On the Optimal Number of Relay Stations in Two-hop Relay Cooperative Cellular Networks

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ABSTRACT
Cooperative communications are adopted as a promising solution to achieve high data rate over large areas in the future 4G wireless system, and the relay station (RS) is the key concept in cooperative communications. However, most existing works in this area focus only on optimal RS selections. In addition, there are only few works consider another crucial issue: how many relay stations we need to place. Only when the number of relay stations is defined, the relay station selection can perform well. In this paper we exploit the Erceg B path loss model to create a formula which describes the impact of varying number of RS on end-to-end channel capacity. In addition to mathematical analysis on the feasibility of the formula, we also examine its performance through a set of simulations. Simulation results verify that the capacity gain of our proposed scheme is promising.

Categories and Subject Descriptors
C.2 [Computer communication networks]: Wireless communications

General Terms
Design, performance

Keywords
cooperative communication; Voronoi tessellation; end-to-end capacity;

1. INTRODUCTION
Wireless signal fading is one of the major problems for the next generation wireless networks, which require high bandwidth efficiency services. Multiple-input multiple-output (MIMO) system is an advanced technology that can effectively exploit the spatial diversity to solve the problem of signal fading and bring significant performance improvements to wireless communication systems. In order to obtain diversity gain in MIMO system, a wireless agent must be set multiple antennas and guarantee at least $\lambda/2$ interval between antennas (λ is wavelength) to avoid signal interference. For example, 2.4 GHz frequency band needs at least 6.125cm interval between antennas. Although the performance of MIMO is quite good, it is suffered from the size limitation of wireless agents. When the size of a wireless agent is small, it may not be able to support multiple transmit antennas.

Cooperative communications have been adopted as another promising solution to achieve high data rate by minimizing the signal fading problem. Compared with MIMO, it is no more limited by the size of wireless agents due to its unique idea. The basic idea is that using multiple distributed transceivers, each with a single antenna can cooperate with one another to form virtual antenna arrays, to achieve some benefits similar to those provided by conventional MIMO systems. As a key concept, a relay station (RS) is the new entity introduced by cooperative communication [1]. Fig. 1 shows the examples of usage scenarios for RS. In the Fig. 1, the RSs are deployed as infrastructure devices without connecting to wired backbone, and serve towards various objectives, such as improving coverage, capacity, or throughput in areas which are not sufficiently covered. When base stations (BSs) and users broadcast signal, the distributed RSs can receive the signal and relay it to relevant users or BSs.

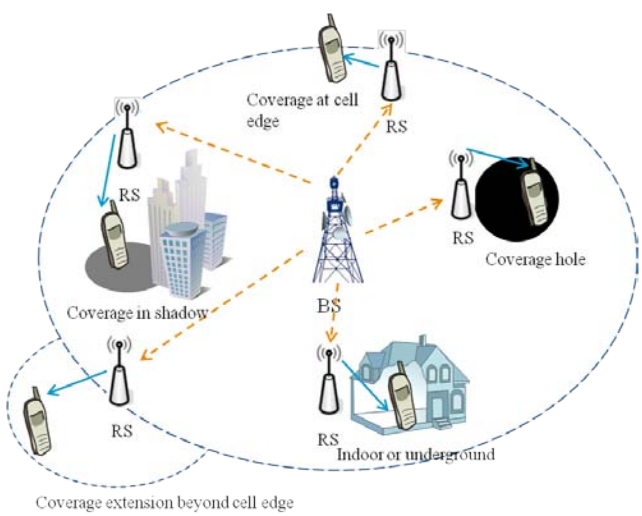

Figure 1. Examples of usage scenarios for RS.

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There are many valuable investigations of practical issues on cooperative communications. Most of these investigations focus on the issue of optimal RS selections and RS assignment. They neglect another important issue related to RS, that is, how many RS we need to place. They just randomly choose the number of RS in their schemes. In fact, the optimal number of RS is one of the important preconditions to guarantee the performance of these schemes.

In this paper, we mathematically analyze the optimal number of RSs for the placement in two-hop relay cellular networks. We use a simple RS selection scheme where the nearest RS to the source is selected. We apply Voronoi tessellation [8] to the cellular environment and derive a formula to analyze the impact of the varying number of RSs on end-to-end channel capacity. Finally, we perform a set of simulations to validate our mathematical analysis.

The rest of the paper is organized as follows. In Section 2, we introduce previous works about RS placement. We then formulate the optimal number of RS placement problem in Section 3. In Section 4, we show that the mathematical analysis is practical through a set of simulations. Finally, Section 5 contains the conclusion of the paper.

2. RELATED WORKS

IEEE 802.16j multi-hop relay network task group, which was created in March 2006, aims to develop low-cost relay infrastructure architectures for IEEE 802.16 systems. In IEEE 802.16j, RSs are effectively utilized in several scenarios such as cell coverage extension, decreasing shadows area, and enhancing channel capacity.

Most of existing works on cooperative communication focus on optimal RS selection [2-4] and RS assignment [5-6]. In [2], a centralized optimal RS selection scheme is proposed. The optimal RS is selected according to a weight value which is calculated by SNR (signal to noise ratio) and latency. In [3], the authors consider both local channel state (SNR) and network topology to select an optimal RS. In [4], both of channel state information at physical layer and queue state information at data link layer are taken into consideration as relay selection criterion. All of these works aim to guarantee the diverse QoS requirements. In addition to RS selection, there also exist some investigations which focus on optimal RS assignments [5~6]. In [5], the authors use the sum rate as a factor and develop a convex optimization problem that provides a tight upper bound. In [6], a linear marking mechanism is used to minimize algorithm complexities. In [7], the author proposed a novel cooperative scheme. A source node using Amplify-and-Forward (AF) RS and Decode-and-Forward (DF) RS together achieves better performance than that of conventional cooperative schemes. Most of these investigations neglect another important issue related to RS, that is, how many RS we need to place. In fact, the optimal number of RS is an important precondition to achieve the cost-effective solution of system deployment to provide reliable communication services.

However, there exist only limited works on optimal RS placements. The works in [9~11] investigate the optimal location of the relay station for the purpose of maximizing the system throughput. In [9], the authors evaluated the impacts of variable distances between BS and RS on system capacity through the simulations. They found that when the users are quite close to BS, the two time-slot transmission scheme for cooperative mode will decrease the capacity, and thus, direct transmission will be better than the cooperative two-hop transmission. In [10], the authors developed a non-linear optimization formulation to calculate the optimal RS position. However, the formula does not effectively describe the relationship between capacity and the number of RS. In [11], the authors characterized the optimal relay location to minimize the outage probability, and then proposed a nearest-neighbor relay assignment scheme where the nearest neighbor to the source is assigned as a relay station. In [12], the authors investigated the system capacity under the varying number of RSs and associated transmit power. The results show that the gain varies from less than 5% to 55% by increasing the number of RSs. In [13], a clustering approach was proposed to solve network planning problems for IEEE 802.16j relay networks. The authors firstly applied a k-means clustering approach to divide the large problem into several small problems. Then the planning problem for each of the clusters was solved independently.

The above investigations just proposed optimal RS locations in cellular, analyzed the relationship based on simulation results. However, none of them accurately analyzed the problem of “How many relay stations we need to deploy”. In fact, this is very important for the network operators to find the cost-effective solution of system deployment.

In this paper, with the goal of deciding the optimal number of RS deployment in two-hop cellular network, we formally derive a formula to analyze the relationship between the number of RS placement and the obtained capacity by the user. We exploit the nearest-neighbor RS selection scheme, which is proposed in [11], to assign a RS to the nearest user. Although the RS selection scheme is straightforward, it has low overhead and similar capacity gain compared with other complicated RS selection schemes. Based on our advocated RS selection scheme, we apply Voronoi tessellation to the cellular environment. Then a cell is divided into several clusters wherein the optimal number of RS deploy problem is solved separately. We calculate the first moment of the number of users and the total link length in a cluster. Based on the calculated results, we develop a formula and use the formula to analyze the impacts of the varying number of RSs on channel capacity. The detailed approach is described in Section 3.

3. THE PROPOSED SCHEME

We consider the downlink transmission of a single cell which contains one BS at the cell center, and multiple destination users. In this paper, we apply Voronoi tessellation to the cellular environment. In Voronoi tessellation [8], there are two Poisson point processes Π_0 and Π_1. Π_1-particle RS_i is chosen to be the closest to Π_0-particle $user_i$ among all the Π_1-particles. Thus the cellular environment will be divided into several clusters. Each cluster consists of a RS at the cluster center and the users which are chosen to be closest to RS and distributed around the RS. Since we use the nearest-neighbor RS selection scheme, the relay link length between RS and user will be the shortest and the user can obtain maximal SNR. Based on Voronoi tessellation, we derive a formula to analyze the relationships between the number of RS placement and the user obtained capacity gain.

To develop a formula for analyzing the impact of varying number of RSs on channel capacity, we make the following assumptions.

- Users are distributed according to a homogeneous spatial Poisson process with intensity ρ_0 .
- RSs are distributed with intensity ρ_1 , and $\rho_1 = m / A$. A and m indicate the cellular areas and the numbers of RS, respectively.
- The processing time in RS is not considered.
- RS operates in time-division model. In the first timeslot, BS broadcasts signal to RS and users and in second timeslot RS retransmits the received signal to users.

Using the idea of Foss [8], the first moment of aggregate characteristics S_f is given as

$$E(S_f) = \rho_0 \int f(x) e^{-\rho_1 \pi |x|^2} dx \, . \tag{1}$$

Taking $f(x) = 1$ and $f(x) = l$, (l is the link length between correspondent) we can get the expectations of the number of users (N_u) and the total link length (L_t) of all users connecting to the nucleus RS in a cluster as Eqs. (2) and (3).

$$E(N_u) = \frac{\rho_0}{\rho_1} \, , \tag{2}$$

$$E(L_t) = \frac{\rho_0}{2\rho_1^{3/2}} \, . \tag{3}$$

According to the investigation in [15], the end-to-end two hop relay capacity is expressed as Eq. (4).

$$C_{e2e} = \left(\frac{1}{C_0} + \frac{1}{C_1}\right)^{-1} = \frac{C_0 C_1}{C_0 + C_1} \tag{4}$$

Then we use the fitted power function $R = \alpha D^\beta$ to express the rate-range relationship [14]. The transmit rate R is the means of channel capacity and D is the distance between correspondents. The one-hop capacity can be expressed as $C_0 = \alpha_0 D_0^{\beta_0}$ and $C_1 = \alpha_1 D_1^{\beta_1}$. C_0 , C_1 , D_0 and D_1 denote the capacity and distance between BS and RS, and between RS and user, respectively. In a real system, the constants α_0 , α_1 , β_0 , and β_1 are known, which include factors for transmitter power, antenna gains, antenna heights, frequency, receiver noise figure, and other factors related with the environment. We substitute $C_0 = \alpha_0 D_0^{\beta_0}$, $C_1 = \alpha_1 D_1^{\beta_1}$ in Eq. (4). Then we can get Eq. (5) as follows.

$$C_{e2e} = \frac{\alpha_0 D_0^{\beta_0} * \alpha_1 D_1^{\beta_1}}{\alpha_0 D_0^{\beta_0} + \alpha_1 D_1^{\beta_1}} \, . \tag{5}$$

Using Eq. (5), we can analyze two-hop relay capacity by D_0 and D_1 . According to Eqs. (2) and (3) which is derived from Voronoi tessellation, we can obtain the average radius of each cluster as follows.

$$E(L_a) = \frac{E(L_t)}{E(N_u)} = \frac{1}{2\sqrt{\rho_1}} \tag{6}$$

Based on previous assumptions $\rho_1 = m / A$, $E(L_a)$ also can be expressed as

$$E(L_a) = \sqrt{\frac{A}{4m}} \tag{7}$$

Lets D_1 equal to $E(L_a)$ which is the average distance between RS and user in a cluster. Substitute $E(L_a)$ for D_1 , we can get expect function (8) as below

$$C_{e2e} = \frac{\alpha_0 D_0^{\beta_0} * \alpha_1 \left(\frac{1}{2\sqrt{\rho_1}}\right)^{\beta_1}}{\alpha_0 D_0^{\beta_0} + \alpha_1 \left(\frac{1}{2\sqrt{\rho_1}}\right)^{\beta_1}} = \frac{\alpha_0 D_0^{\beta_0} * \alpha_1 \left(\sqrt{\frac{A}{4m}}\right)^{\beta_1}}{\alpha_0 D_0^{\beta_0} + \alpha_1 \left(\sqrt{\frac{A}{4m}}\right)^{\beta_1}} \tag{8}$$

Now we make clear the relationship between two-hop relay capacity and the number of placement RS. Eq. (8) describes the distance between BS and RS, and the numbers of RSs impacting on the end-to-end capacity. In this paper we fix D_0 which is the distance between BS and RS, and use Eq. (8) to analyze the optimal numbers of RSs to guarantee end-to-end capacity that users can obtain in each cluster. In addition, we compare the mathematical analysis based on Eq. (8) with simulation results.

4. SIMULATION RESULTS

In this paper, we use the Erceg path loss model [16]. The model covers three suburban terrain categories, including Erceg A, Erceg B, and Erceg C [16]. Terrain category A is hilly terrain with moderate-to-heavy tree densities. Terrain category C is mostly flat terrain with light tree densities. Terrain category B is between A and C. Table 1 exhibits the relationship between transmission mode and distance in Erceg B model. Based on Table 1, when RS operates in time-division model, a two-hop link achieve maximal end-to-end rate 11.439Mbps. Thus, if direct transmission between BS and user is larger than 5.719(11.439/2)Mbps, there is no capacity gain when applying two-hop cooperative communication. Due to this reason, based on Table 1, if the one-hop capacity between BS to user is less than 7.623Mbps, the user use two-hop relay transmit mode to obtain cooperative communication gains. In this reason, we distribute RS in range of 1.7km to provide service for users who are located in the range from 1.7km to 3.2km as Fig. 2.

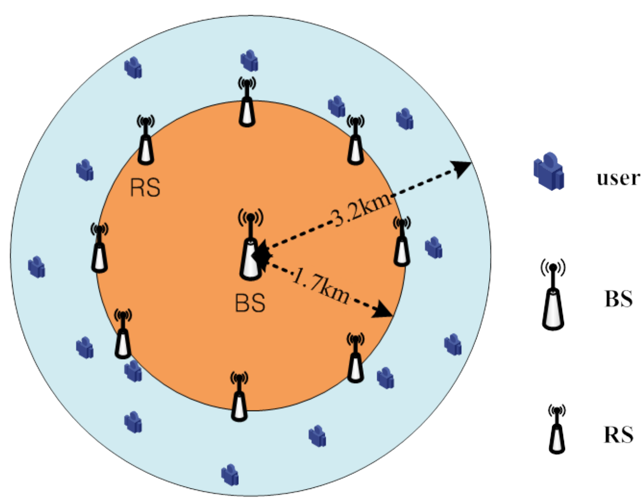

Figure 2. Simulation environment.

In our simulation, the RS support cooperative communication services for users who are located in the cluster range of RS. Based on [10], the fitted parameters as follows:

$\alpha_0 = 7.638$, $\beta_0 = -0.665$, $\alpha_1 = 16.01$, $\beta_1 = -1.706$.

Table 1. The relationship between transmission mode and distance in Erceg B model [10]

Modulation mode	Rate (Mbps)	Transmit range (km)
BPSK 1/2	1.269	3.2
QPSK 1/2	2.538	2.7
QPSK 3/4	3.816	2.5
16-QAM 1/2	5.085	1.9
16-QAM 3/4	7.623	1.7
64-QAM 2/3	10.161	1.3
64-QAM 3/4	11.439	1.2

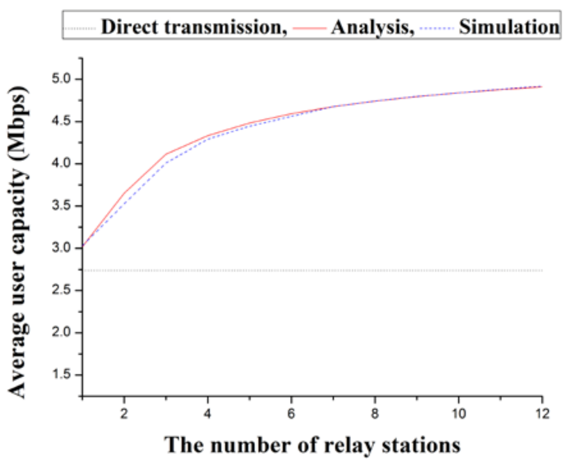

Figure 3. Numerical analysis and simulation results.

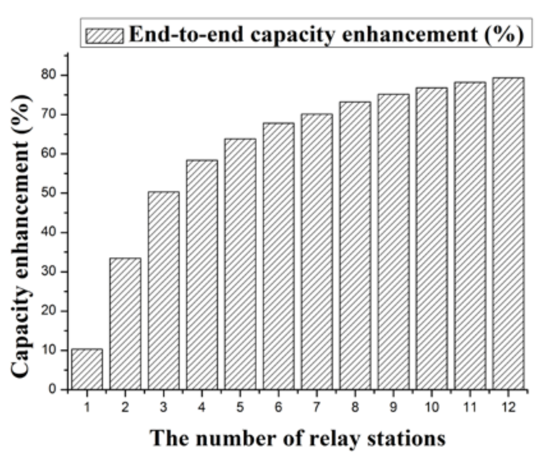

Figure 4. End-to-end capacity enhancement.

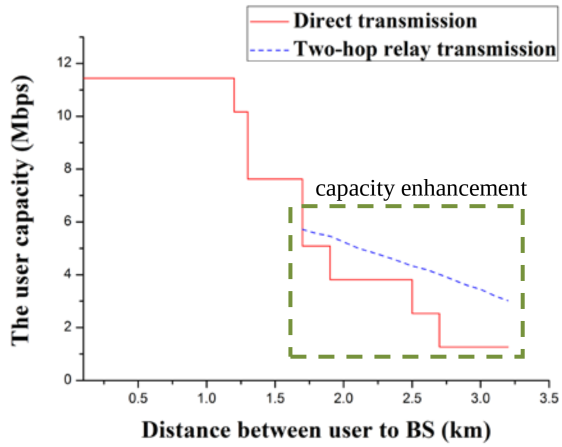

Figure 5. Comparisons between the direct transmit capacity and two relay transmit capacity.

Fig. 3 shows the numerical results and comparisons with simulations. After the user is established with a certain cluster in RS, we evaluate the obtained capacity for the user. It shows that the graph of numerical results is very similar to graph of simulations. It indicates that our proposed formula can reliably express the impact of varying number of RS on end-to-end channel capacity. In Fig 4, the user at the range from 1.7km to 3.2 can obtain 2.737Mbps average capacity with direct transmission. With two-hop cooperative transmission, the capacity has been enhanced 79% when we deploy 12 RSs.

In Fig. 5, we compare the capacity between direct transmission and two-hop relay transmission, when 12 RSs are distributed in range of 1.7km and the distance between user and BS is increasing. Fig 5 shows there is no cooperative communication gain when the user is located in the range of 1.7km. This is because the channel state between BS and user is good enough for direct transmission and the two time-slot transmission scheme of cooperative transmission decreases the capacity. In the range from 1.7km to 3.2km, the user can obtain expected capacity gain.

5. CONCLUSION

Deciding the optimal number of relay station (RS) is an important issue in the area of cooperative communication. In this paper, we mathematically develop a formula to describe the impacts of varying number of RS on end-to-end channel capacity.

We divide the cell into several clusters based on Voronoi tessellation, and then analyze the relationship between distributed number of RS and the average cluster range. It is an effective scheme to RS selection and helpful to calculate the expectation of each cluster range. Then we develop a formula to analyze the relationship between the number of RS placement and the obtained end-to-end capacity by the user. Finally, a set of simulation results show that our numerical analysis is reliable.

6. ACKNOWLEDGMENTS

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